Generalized p-Persistent CSMA for Asynchronous Multiple-Packet Reception

Yijin Zhang, *Senior Member, IEEE*, Aoyu Gong, Yuan-Hsun Lo, *Member, IEEE*, Jun Li, *Senior Member, IEEE*, Feng Shu, *Member, IEEE*, and Wing Shing Wong, *Fellow, IEEE*

***Abstract*—This paper considers a multiple-access system with multiple-packet reception (MPR) capability γ, i.e., a packet can be successfully received as long as it overlaps with $\gamma - 1$ or fewer other packets at any instant during its lifetime. To efficiently utilize the MPR capability, this paper generalizes p-persistent carrier-sense multiple access (CSMA) to consider that a user with carrier sensing capability c adopts the transmission probability p_n if this user has sensed n ongoing transmissions for $n = 0, 1, \ldots, c-1$. This paper aims to model the characteristics of such CSMA and to design transmission probabilities for achieving maximum saturation throughput. To this end, we first formulate such CSMA as a parameterized Markov decision process (MDP) and use the long-run average performance to evaluate the saturation throughput. Second, by observing that the exact values of optimal transmission probabilities are in general infeasible to find, we modify this MDP to establish an upper bound on the maximum throughput, and modify this MDP again to propose a heuristic design with near-optimal performance. Simulations with respect to a wide range of configurations are provided to validate our study. The throughput performance under more general models and the robustness of our design are also investigated.**

***Index Terms*—Access protocols, multiple-packet reception, performance analysis, Markov decision process**

I. Introduction

A. Motivation

In a random access system, the role of *medium access control* (MAC) mechanism is to allow a set of uncoordinated users to efficiently share the same communication channel. On account of their distributed nature and inherent flexibility, *carrier-sense multiple access* (CSMA)-type protocols have been the foundation of random access based MAC for many communications standards. By the help of carrier sensing, they are able to support variable packet lengths and provide significant performance improvement over the pioneering ALOHA-type protocols.

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Y. Zhang is with the School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China, and also with the State Key Laboratory of Integrated Services Networks (Xidian University), Xi'an 710071, China. E-mail: yijin.zhang@gmail.com.

A. Gong, J. Li and F. Shu are with the School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China. E-mail: gongaoyu@gmail.com; {jun.li, shufeng}@njust.edu.cn.

Y.-H. Lo is with the Department of Applied Mathematics, National Pingtung University, Pingtung, Taiwan 90003. E-mail: yhlo0830@gmail.com.

W. S. Wong is with the Department of Information Engineering, The Chinese University of Hong Kong, Shatin, N. T., Hong Kong. E-mail: wswong@ie.cuhk.edu.hk.

The p-persistent CSMA protocol [1], one of the original versions of CSMA, was proposed by Kleinrock and Tobagi in 1975. For the following three reasons, it has drawn much research interest during recent decades. First, the p-persistent CSMA simply allows a user to begin a packet transmission with a common probability p whenever this user has sensed channel idle, and thus is well-suited for implementation. Second, the p-persistent CSMA is analytically tractable due to its memoryless nature in access. Third, the behaviors of many sophisticated CSMA-type protocols can be closely approximated by p-persistent CSMA (at least from the standpoint of maximum throughput) if the p value is selected to guarantee that the same average backoff interval is used [2], [3]. This correspondence suggests that it is essential to derive the optimal p for achieving the maximum throughput in p-persistent CSMA.

Early studies on p-persistent CSMA assumed the *single-packet reception* (SPR) model, which allows a packet to be successfully received only when it does not overlap with another on the channel. Bianchi [2] derived the saturation throughput of p-persistent CSMA. Calì *et al.* [3] and Bononi *et al.* [4] used the balance between collision durations and idle times to approximate the optimal p for maximizing the saturation throughput. Bruno *et al.* [5] formally proved this approximation is asymptotically exact.

Recently, the SPR assumption has become less relevant due to the advance of *multiple-packet reception* (MPR) techniques [6], [7] that enable successful receptions of time-overlapping packets at the physical layer. One such example is *multi-user multiple-input multiple-output* (MU-MIMO) that has been supported in IEEE 802.11ac; an evolutionary overview of this technique can be found in [8]. In this paper, we restrict our attention to a specific type of MPR, namely γ-MPR, which signifies the assumption that a packet can be successfully received as long as it overlaps with $\gamma-1$ or fewer other packets at any instant during its lifetime. Here, γ refers to the *MPR capability*. To convert the MPR capability to the MAC throughput as efficiently as possible, it is desirable to gain a clear insight into the impact of γ-MPR on the analysis and design of p-persistent CSMA.

With this goal in mind, Zhang *et al.* [9] showed that the maximum saturation throughput per unit cost increases with γ, Bae *et al.* [10] dealt with the optimal p for maximizing the saturation throughput under γ-MPR, and Chan *et al.* [11] derived the maximum decentralized stable throughput. However, it has been pointed out in [12] that p-persistent CSMA *cannot* be used to efficiently utilize the γ-MPR channel. The reason is that p-persistent CSMA operates in a synchronous mode, i.e., it allows a new transmission to begin only after the completion of ongoing transmissions. This restriction leads to bandwidth waste when the number of ongoing transmissions is smaller than γ. Clearly, such waste would become severer if different transmissions last for different durations.

To combat this weakness, Chan *et al.* [12] proposed an extension of p-persistent CSMA that operates in an asynchronous mode, called XL-CSMA. In XL-CSMA, a user is able to detect if the number of ongoing transmissions is $0, 1, \ldots, c-1$ or $\geq c$, and is allowed to adopt different transmission probabilities according to the sensed number of ongoing transmissions. Here, we refer to c as *carrier sensing capability*. However, this work did not provide a general analytical model for characterizing XL-CSMA, and the setting of transmission probabilities in XL-CSMA is far from the optimum, which will be shown in Section VIII. Moreover, the design of XL-CSMA assumes $c = \gamma$, which is usually unrealistic in distributed wireless networks with hardware cost constraints.

Inspired by the insights in [12], this paper focuses on the following fundamental questions.

(i) By taking into account an arbitrary carrier sensing capability, how to design a generalization of p-persistent CSMA under γ-MPR and how to implement it in IEEE 802.11-like networks?

(ii) How to analyze the saturation throughput of such CSMA under γ-MPR?

(iii) How to design the optimal transmission probabilities for maximizing the saturation throughput of such CSMA under γ-MPR?

Note that these questions have been investigated in [3]–[5] for SPR (i.e., $\gamma = 1$), and also have been investigated in [9], [10], [12] when each user only can sense channel idle or busy (i.e., $c = 1$).

B. Contributions

The contributions of this paper are summarized as follows.

(i) We generalize traditional p-persistent CSMA to consider that a user with the carrier sensing capability c adopts the transmission probability p_n if this user has sensed n ongoing transmissions at the beginning of a generic slot for $n = 0, 1, \ldots, c-1$, and implement such CSMA in IEEE 802.11-like networks using c backoff processes.

(ii) We formulate the generalized p-persistent CSMA as a parameterized *Markov decision process* (MDP) and use the long-term average reward to evaluate the saturation throughput.

(iii) By observing that the exact values of optimal transmission probabilities are in general infeasible to find, we modify this MDP to establish an upper bound on the maximum throughput, and then modify this MDP again to propose a heuristic design with near-optimal performance.

(iv) We extend the throughput analysis to more general models, and show that our heuristic design is robust to them in many scenarios.

Our work to be presented in this paper can include the studies in [3]–[5], [9], [10], [12] as particular cases.

C. Other Related Work

Motivated by [12], many recent literatures have been dedicated to the design and analysis of asynchronous CSMA/CA for better utilizing the γ-MPR channel. A comprehensive survey of them can be found in [13].

Babich *et al.* [14] modified the IEEE 802.11 CSMA/CA protocol to allow a user to decrease its backoff counter as soon as the number of sensed ongoing transmissions is below a threshold. Mukhopadhyay *et al.* [15] proposed a solution to address the ACK-delay problem, which was not considered in [14]. In their solution, a user is required to freeze its backoff counter once the number of ongoing transmissions exceeds $\gamma - 1$ and resume decrementing its backoff counter only when the channel becomes idle again. Wu *et al.* [16] further modified the scheme in [14] to require that all overlapping transmissions are completed simultaneously (by packet fragmentation or aggregation) even if they are begun at different time instants. One of our prior works [17] modified the IEEE 802.15.4 CSMA/CA protocol to allow a user to decrease its backoff counter at every slot boundary and begin a packet transmission if this user has sensed fewer than a certain number of ongoing transmissions at the end of backoff. It will be shown in Section III that the generalized p-persistent CSMA considered in this paper can also be converted to a CSMA/CA scheme, and will be shown in Section VIII that our design enjoys better throughput performance than state-of-the-art schemes.

For these asynchronous CSMA/CA, the analytical models developed in [14], [16]–[18] are based on Markov processes, while the one developed in [15] is based on a renewal-theoretic fixed-point analysis. It should be noted that the models in [14], [17] share some similarities with the one to be presented in this paper, as [14], [17] studied CSMA/CA using the behavior of p-persistent CSMA. However, [14], [17] assumed that the transmission probabilities are the same if the sensed number of ongoing transmissions is below a threshold, which is more restrictive than the general setting considered in this paper.

To our best knowledge, there is no previously known theoretical investigation on the optimal access parameters for asynchronous CSMA-type protocols under MPR.

The remainder of this paper is organized as follows. In Section II, we set up the system model. Section III describes the design of the generalized p-persistent CSMA and how to convert it to CSMA/CA. In Section IV, we formulate such CSMA as an MDP to evaluate the saturation throughput. Built on some modifications on this MDP, an upper bound on the maximum throughput is provided in Section V, and a heuristic design with near-optimal performance is proposed in Section VI. The throughput analysis under more general modes is discussed in Section VII. In Section VIII, we present simulations to verify the studies in Sections IV–VII and compare our design against other schemes. Section IX concludes this paper.

II. System Model

We consider a wireless network, where a finite number, $N \geq 2$, of infinitely backlogged users contend to transmit packets to a common receiver. All the users are within the reception range of the receiver and are within the interference range of each other. The time axis is divided into time slots of equal duration and the slot boundaries are known to the users.

We assume that each user can perform carrier sensing to detect if the number of ongoing transmissions is $0, 1, \ldots, c-1$ or $\geq c$, and the time required to do so is negligible compared with the slot length. Such enhanced carrier sensing can be implemented if each user is equipped with an array of antennas [14], [15], [19]. The signal processing algorithms [20] involved in this case usually follow the eigenvalue analysis of the correlation matrix, and hence their computational burden is acceptable in some kinds of distributed networks. Moreover, the current antenna miniaturization technology [21] is able to produce acceptable-size arrays of 2.4G/5.8Ghz antennas. Nevertheless, taking into account the difficulty in implementation, this paper will consider all possible values of c.

By assuming negligible wireless fading effect, we consider that the receiver has the γ-MPR capability, i.e., can recover all n signals simultaneously transmitted in a slot if $1 \leq n \leq \gamma$ and recover none of them otherwise. Furthermore, by assuming that channel coding is not used to protect the packets, we consider that a packet can be successfully received if it overlaps with $\gamma-1$ or fewer other packets at any instant during its transmission lifetime, and is discarded otherwise. To avoid trivial cases, we consider $1 \leq \gamma < N$.

The receiver sends an acknowledgement (ACK) on a separate narrow feedback channel immediately after the end of every successful transmission; if multiple successful transmissions are completed simultaneously, the receiver sends an aggregated ACK. If the transmission outcome is unsuccessful, the corresponding sender is required to retransmit the same packet at the next transmission within the retry limit. We also assume that the ACK reception time is negligible compared with the slot length, as in [1], [5], [12], [14].

For MAC analysis under different types of services, the packet lengths in units of slot (i.e., the service times of agents in queuing theory) are often modeled as exponentially distributed variables. In this paper, to simplify the presentation, we consider that the packet lengths always correspond to integer numbers of slots, and thus assume they follow the geometric distribution with average value $\Lambda > 1$, which is a discrete analogue of the exponential distribution. More specifically, for $\lambda = 1, 2, \ldots$, the probability that a packet has a length equal to λ slots is $(1/\Lambda)(1-1/\Lambda)^{\lambda-1}$.

To maintain the analytical tractability of the problem, following [3]–[5], [14], we assume that successive retransmissions of the same packet are independent although they obviously have the same length. As such, if there is an ongoing transmission at a slot, this transmission will be completed at the end of this slot with probability $1/\Lambda$. The accuracy of this assumption will be examined via simulations in Section VIII.

In this paper, we will mainly focus on the aforementioned system model. The extension to more general models will be discussed in Section VII.

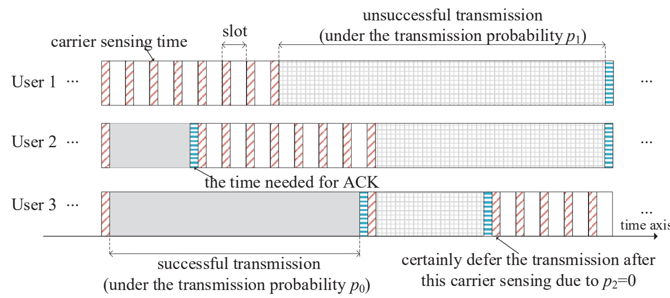


Fig. 1. An example of the working procedure of the generalized p-persistent CSMA for the case of $N = 3$ and $c = \gamma = 2$.

Remark 1: The aforementioned system model with $c = \gamma = 1$ is exactly the model considered in [5] for studying traditional p-persistent CSMA under SPR. Such consistency allows us to examine the effect of MPR capability and carrier sensing capability on the performance improvement.

III. Protocol Description

To comply with the γ-MPR capability, we assume $1 \leq c \leq \gamma$ and consider a natural generalization of the traditional p-persistent CSMA. That is, each silent user is required to perform carrier sensing at the beginning of every slot; if a silent user detects $n \leq c-1$ ongoing transmissions at the beginning of a slot, this user will begin a transmission with probability $0 \leq p_n < 1$, otherwise, this user will begin a transmission with probability $p_n = 0$. To avoid that all users will always keep silent once the channel has become idle, it is natural to require $0 < p_0 < 1$. We further define the following parameter vector:

$$\mathbf{p} \triangleq (p_0, p_1, \ldots, p_{c-1}),$$

with the value domain $\mathbb{D} \triangleq \mathbb{D}_0 \times \mathbb{D}_1 \times \cdots \times \mathbb{D}_{c-1}$, where $\mathbb{D}_0$ is the real-number interval $(0, 1)$, $\mathbb{D}_n$ is the real-number interval $[0, 1)$ for $1 \leq n \leq c-1$, and $\times$ is the Cartesian product. In addition, for the convenience of the following analysis, although it is impossible for a user to sense N ongoing transmissions, we assume that a user will adopt the transmission probability $p_N = 0$ if there are N ongoing transmissions.

An example of the working procedure of the generalized p-persistent CSMA for the case of $N = 3$ and $c = \gamma = 2$ is illustrated in Fig. 1.

To convert the generalized p-persistent CSMA to a CSMA/CA scheme for IEEE 802.11-like networks, we require each user to maintain contention window n with the constant size $W_n = \lfloor 2/p_n - 1 \rceil$ if $p_n > 0$ for $n = 0, 1, \ldots, c-1$. Here, $\lfloor x \rceil$ denotes the integer nearest to x. For each such contention window n, we further require a user to maintain backoff counter b_n with an initialized value that is an integer drawn uniformly from the range $[0, W_n - 1]$, and to perform the following steps as soon as this user senses n ongoing transmissions at the carrier sensing phase of a slot.

(i) If $b_n > 0$, decrease b_n by one.

(ii) Otherwise, immediately send a packet and reset b_n as an integer drawn uniformly from the range $[0, W_n - 1]$.

IV. Analytical Modeling

Given the number of users N, the average packet length Λ, the MPR capability γ and the carrier sensing capability c, we define the *network throughput* $R(\mathbf{p})$ as the long-term average successfully transmitted packet lengths of all users per slot when the parameter vector $\mathbf{p}$ is adopted. Obviously, we have $0 \le R(\mathbf{p}) \le \gamma$ for any $\mathbf{p} \in \mathbb{D}$.

In this section, we formulate the generalized p-persistent CSMA under γ-MPR as a parameterized MDP [22], [23], and use the long-term average reward of this MDP to theoretically evaluate $R(\mathbf{p})$.

A. MDP Formulation

Under the generalized p-persistent CSMA, the ongoing transmissions at a generic slot are composed of two disjoint sets: the ongoing transmissions in the carrier sensing phase and the ongoing transmissions that are begun after the carrier sensing phase (also called new transmissions at this slot).

Consider a state process $(X_t)_{t\in\mathbb{N}}$, where $X_t \in \{0, 1, \dots, N\}$ denotes the number of ongoing transmissions in the carrier sensing phase of slot t. From the descriptions in Sections II and III, we see that each silent user independently begins a new transmission with probability p_{X_t} after carrier sensing at slot t, while every ongoing transmission at slot t is independently completed at the end of slot t with probability $1/\Lambda$. This observation indicates that the probability of moving to the next state in $(X_t)_{t\in\mathbb{N}}$ depends only on the present state and not on the previous states. Thus, $(X_t)_{t\in\mathbb{N}}$ can be viewed as a discrete-time Markov chain with the finite state space $\mathcal{S} \triangleq \{0, 1, \dots, N\}$.

Based on this discrete-time Markov chain, we define a parameterized MDP, $\mathcal{M}$, by introducing the following definitions of actions, parameterized policy, state transition probabilities and reward.

(i) *Actions:* If action a is chosen at state n, it means that a users begin new transmissions after carrier sensing when the state is n. Let $\mathcal{A}_n$ denote the set of all possible actions when the state is n. Since $p_n = 0$ when $c \le n \le N$, we find

$$\mathcal{A}_n = \begin{cases} \{0, 1, \dots, N-n\} & \text{if } 0 \le n \le c-1, \\ \{0\} & \text{otherwise.} \end{cases}$$

(ii) *Parameterized policy:* We define a parameterized policy that chooses action a at state n with probability $\mu_{n,a}(\mathbf{p})$ for each $n \in \mathcal{S}$ and each $a \in \mathcal{A}_n$. If action a is chosen at state n, it means that a out of $N-n$ users actually begin to transmit after carrier sensing when these $N-n$ users independently begin to transmit with probability p_n. So, we have

$$\mu_{n,a}(\mathbf{p}) = \binom{N-n}{a} p_n^a (1-p_n)^{N-n-a}, \quad \forall n \in \mathcal{S}, \forall a \in \mathcal{A}_n. \tag{1}$$

(iii) *State transition probabilities:* We define the state transition probabilities by

$$\beta_{n,n'}(\mathbf{p}) \triangleq \Pr(x_{t+1} = n' | x_t = n, \mathbf{p}), \quad \forall n, n' \in \mathcal{S}.$$

For a transition from $X_t = n$ to $X_{t+1} = n'$, if action a is chosen, it is required that a should be not smaller than $\max(0, n'-n)$ but not larger than $N-n$, and $n+a-n'$ out of $n+a$ users complete their transmissions at the end of slot t. Then, we have

$$\beta_{n,n'}(\mathbf{p}) = \sum_{a=\max(0,n'-n)}^{N-n} \mu_{n,a}(\mathbf{p}) \binom{n+a}{n+a-n'} \left(\frac{1}{\Lambda}\right)^{n+a-n'} \left(1-\frac{1}{\Lambda}\right)^{n'}, \quad \forall n, n' \in \mathcal{S}. \tag{2}$$

(iv) *Reward:* The reward at state n, denoted by $r_n(\mathbf{p})$, is defined as the average of total packet lengths of all the successful transmissions that are begun when the state is n. We further define $r_{n,a}(\mathbf{p})$ as the reward that is gained when action a is chosen at state n. So, we have

$$r_n(\mathbf{p}) = \sum_{a\in\mathcal{A}_n} \mu_{n,a}(\mathbf{p}) r_{n,a}(\mathbf{p}), \quad \forall n \in \mathcal{S}. \tag{3}$$

In the following subsection, we will present an evaluation of $r_{n,a}(\mathbf{p})$ to complete the formulation of $\mathcal{M}$.

B. Reward Evaluation

Since that the silent users are allowed to begin transmissions even if the channel is found busy, the success of a λ-slot transmission not only depends on the interference of other transmissions at its first transmission slot, but also depends on the interference during its remaining $\lambda - 1$ transmission slots. Due to this property, we use $q_{\lambda,h_1}(\mathbf{p})$ to denote the success probability of a λ-slot transmission with h_1 other ongoing transmissions in its first transmission slot. By taking into account all possible packet lengths, we have

$$r_{n,a}(\mathbf{p}) = a \sum_{\lambda=1}^{\infty} \frac{\lambda}{\Lambda} \left(1-\frac{1}{\Lambda}\right)^{\lambda-1} q_{\lambda,n+a-1}(\mathbf{p}), \quad \forall n \in \mathcal{S}, \forall a \in \mathcal{A}_n. \tag{4}$$

It remains only to evaluate $q_{\lambda,h_1}(\mathbf{p})$ for all possible λ and h_1. To this purpose, by assuming a given packet is being transmitted in both the present and next slots, we use $\xi_{h,h'}(\mathbf{p})$ to represent the transition probability that there are h' other ongoing transmissions in the next slot when there have been h other ongoing transmissions in the present slot. For this transition, if j out of these h other transmissions are completed at the end of present slot, we know $h'-h+j$ out of $N-1-h+j$ silent users will begin their transmissions in the next slot. Thus, for $0 \le h \le N-1$ and $0 \le h' \le N-1$, we have

$$\xi_{h,h'}(\mathbf{p}) = \sum_{j=0}^{h} \binom{h}{j} \left(\frac{1}{\Lambda}\right)^j \left(1-\frac{1}{\Lambda}\right)^{h-j} \binom{N-1-h+j}{h'-h+j} p_{h-j+1}^{h'-h+j} (1-p_{h-j+1})^{N-1-h'}. \tag{5}$$

Let $g_{m,h_1,h_m}(\mathbf{p})$ describe the probability of having h_m other ongoing transmissions in the m-th slot of a given transmission and having fewer than γ other ongoing transmissions in each of the first $m-1$ slots of this given transmission, provided that there are h_1 other ongoing transmissions in the first slot of this given transmission. Obviously, $g_{1,h_1,h_1}(\mathbf{p}) = 1$. Then, for $m = 2, \dots, \lambda$, we can recursively obtain

$$g_{m,h_1,h_m}(\mathbf{p}) = \sum_{h_{m-1}=0}^{\gamma-1} g_{m-1,h_1,h_{m-1}}(\mathbf{p})\xi_{h_{m-1},h_m}(\mathbf{p}). \quad (6)$$

As a λ-slot transmission is successful if and only if there are fewer than γ other ongoing transmissions in each of the λ slots of this transmission, we obtain

$$q_{\lambda,h_1}(\mathbf{p}) = \sum_{h_\lambda=0}^{\gamma-1} g_{\lambda,h_1,h_\lambda}(\mathbf{p}). \quad (7)$$

Therefore, $r_{n,a}(\mathbf{p})$ can be computed for each $n \in \mathcal{S}$ and each $a \in \mathcal{A}_n$ using Eqs. (4)–(7).

C. Average Reward

As $p_0 > 0$, we from Eqs. (1) and (2) observe that for every $\mathbf{p} \in \mathbb{D}$, it is possible to go from every state to every state in the Markov chain $(X_t)_{t\in\mathbb{N}}$ with positive probability (not necessarily in one move). This implies that the parameterized MDP $\mathcal{M}$ is *ergodic* for every $\mathbf{p} \in \mathbb{D}$.

Due to the ergodic property, $\mathcal{M}$ has a long-term average reward for every $\mathbf{p} \in \mathbb{D}$, defined by

$$\lim_{T\to\infty} \frac{1}{T} E_{\mathbf{p}}\Big[\sum_{t=0}^{T-1} r_{x_t}(\mathbf{p})\Big], \quad (8)$$

where $E_{\mathbf{p}}$ denotes the expectation taken with respect to the distribution of $\mathcal{M}$ with the parameters $\mathbf{p}$. From the MDP formulation in Section IV.A, we see that the network throughput $R(\mathbf{p})$ is exactly the long-term average reward defined in (8).

Still due to the ergodic property, $R(\mathbf{p})$ is independent of the initial state. Furthermore, the steady state probability of state n, $\pi_n(\mathbf{p})$, is unique for every $\mathbf{p} \in \mathbb{D}$. We also from (4) see $r_n(\mathbf{p}) = 0$ when $n \geq c$. As such, $R(\mathbf{p})$ can be calculated by

$$R(\mathbf{p}) = \sum_{n=0}^{c-1} \pi_n(\mathbf{p}) r_n(\mathbf{p}), \quad (9)$$

where $\pi_0(\mathbf{p}), \pi_2(\mathbf{p}), \dots, \pi_{c-1}(\mathbf{p})$ can be obtained by the balance equations:

$$\sum_{n\in\mathcal{S}} \pi_n(\mathbf{p})\beta_{n,n'}(\mathbf{p}) = \pi_{n'}(\mathbf{p}), \qquad \forall n' \in \mathcal{S}. \quad (10)$$

Remark 2: Our analytical model reduces to the model presented in [3]–[5] for analyzing the traditional p-persistent CSMA under SPR if we set $\gamma = 1$, and reduces to the model presented in [9], [10], [12] for analyzing the traditional p-persistent CSMA under γ-MPR if we set $c = 1$.

D. Discussions

Following the above analysis, a natural design goal is to find the optimal $\mathbf{p}$ that maximizes $R(\mathbf{p})$, i.e.,

$$R_{max} = \max_{\mathbf{p}\in\mathbb{D}} R(\mathbf{p}), \quad \mathbf{p}_{opt} = \arg\max_{\mathbf{p}\in\mathbb{D}} R(\mathbf{p}). \quad (11)$$

To solve this problem for a general network configuration with $1 \leq c \leq \gamma$, a traditional thought is to use gradient-based methods [22] that have been widely applied to optimize the parameterized policy of Markov systems. However, it is difficult to calculate the gradient of $R(\mathbf{p})$ when $1 < c \leq \gamma$, since that, as shown in Eqs. (4)–(7), the term $q_{\lambda,h_1}(\mathbf{p})$ in $r_n(\mathbf{p})$ is obtained recursively and the number of recursive steps increases with the geometrically distributed packet length.

On the other hand, a policy iteration type algorithm was proposed in [23] to optimize a special category of parameterized MDP, which satisfies both of the following two conditions:

(i) $C1$: the state space can be partitioned such that the transition probabilities and rewards of all states in each partition are not affected by any parameter in $\mathbf{p}$ or are affected by a distinct parameter in $\mathbf{p}$.
(ii) $C2$: the change of the parameters in $\mathbf{p}$ does not affect the conditional probability of the state value when this state is known to be included in a partition.

Unfortunately, $\mathcal{M}$ with $1 < c \leq \gamma$ does not belong to this category, due to the fact that, as shown in Eqs. (3)–(7), the reward $r_0(\mathbf{p})$ is affected by all the parameters in $\mathbf{p}$ and the reward $r_n(\mathbf{p})$ for $1 \leq n \leq c-1$ is affected by all the parameters in $\mathbf{p}$ except p_0.

The aforementioned difficulties in obtaining $\mathbf{p}_{opt}$ motivate us to modify $\mathcal{M}$ such that an upper bound on R_{max} can be efficiently found, and to propose a heuristic design that can achieve the throughput performance close to this upper bound. These two issues will be dealt with in Sections V and VI, respectively. Moreover, it will be shown in Section VIII that our heuristic design and the design obtained from "GlobalSearch" in the MATLAB Global Optimization Toolbox enjoy almost the same throughput performance.

V. An Upper Bound on Maximum Throughput

In this section, we consider a parameterized MDP modified from $\mathcal{M}$ and apply the policy iteration proposed in [23] to efficiently find an upper bound on R_{max}.

A. MDP Modification

We modify $\mathcal{M}$ to $\mathcal{M}^*$ by only redefining the reward at each state. Here, the reward at state n of $\mathcal{M}^*$, denoted by $r_n^*(\mathbf{p})$, is defined as the average of total packet lengths of the transmissions, each of which is begun when the state is n and overlaps with $\gamma - 1$ or fewer other packets at the first slot of its transmission. By this definition, we have

$$r_n^*(\mathbf{p}) = \Lambda \sum_{a=0}^{\gamma-n} a \cdot \mu_{n,a}(\mathbf{p}), \quad \forall n \in \mathcal{S}. \quad (12)$$

As $\mathcal{M}$ and $\mathcal{M}^*$ share the same state space and the same transition probabilities, $\mathcal{M}^*$ is ergodic as well. Hence, similar to Eq. (9), the long-term average reward of $\mathcal{M}^*$, denoted by $R^*(\mathbf{p})$, can be calculated as

$$R^*(\mathbf{p}) = \sum_{n=0}^{c-1} \pi_n(\mathbf{p}) r_n^*(\mathbf{p}). \quad (13)$$

By comparing Eqs. (3), (4), (9), (12) and (13), we see

$$R^*(\mathbf{p}) \geq R(\mathbf{p}), \quad \forall \mathbf{p} \in \mathbb{D}. \quad (14)$$

Hence, to provide an upper bound on the maximum network throughput R_{max}, we formulate the following optimization problem:

$$R_{upp} = \max_{\mathbf{p}\in\mathbb{D}} R^*(\mathbf{p}), \quad \mathbf{p}_{upp} = \arg\max_{\mathbf{p}\in\mathbb{D}} R^*(\mathbf{p}). \tag{15}$$

B. Policy Iteration Algorithm

As $\mathcal{M}^*$ is ergodic, the relative value due to starting from state n of $\mathcal{M}^*$ can be defined as

$$v_n^*(\mathbf{p}) \triangleq E\Big\{\sum_{t=0}^{\infty} r_{x_t}^*(\mathbf{p}) - R^*(\mathbf{p})\Big| x_0 = n\Big\}, \quad \forall n \in \mathcal{S}. \tag{16}$$

Furthermore, for each given $\mathbf{p} \in \mathbb{D}$, all $v_n^*(\mathbf{p})$s satisfy the following relative value Bellman equation [24]:

$$v_n^*(\mathbf{p}) = r_n^*(\mathbf{p}) - R^*(\mathbf{p}) + \sum_{n'\in\mathcal{S}} \beta_{n,n'}(\mathbf{p}) v_{n'}^*(\mathbf{p}), \forall n \in \mathcal{S}. \tag{17}$$

From the description of $\mathcal{M}^*$ in Section V.A, we see that the transition probabilities and reward of state n are only affected by the parameter p_n if $0 \le n \le c-1$ and are unaffected by any parameter in $\mathbf{p}$ otherwise. Hence, $\mathcal{M}^*$ can be partitioned to satisfy both of the conditions $C1$ and $C2$ introduced in Section IV.D. Owing to this property, we simplify the policy iteration algorithm proposed in [23] to comply with the specific formulation of our problem. The procedure is based on the relative value Bellman equation (17) and is summarized in Algorithm 1 below. Interested readers are referred to [23] for the convergence proof.

Algorithm 1 Policy iteration algorithm to find $\mathbf{p}_{upp}$.

1: **Initialization**. Choose an arbitrary parameter vector $\mathbf{p}^{(0)} \in \mathbb{D}$ as the initial value and set $k = 0$.

2: **Evaluation.** Calculate the relative value $v_n^*(\mathbf{p}^{(k)})$ for each $n \in \mathcal{S}$ by Eq. (17).

3: **Improvement.** Update the new parameter as

$$p_n^{(k+1)} = \arg\max_{\widetilde{p_n}\in\mathbb{D}_n} r_n^*(\widetilde{p_n}) + \sum_{n'\in\mathcal{S}} \beta_{n,n'}(\widetilde{p_n}) v_{n'}^*(\mathbf{p}^{(k)}) \tag{18}$$

for $n = 0, 1, \ldots, c-1$. If there exists $\widetilde{p_n} = p_n^{(k)}$ that attains the maximum in Eq. (18), we set $p_n^{(k+1)} = p_n^{(k)}$.

4: **Stopping Rule.** If $\mathbf{p}^{(k+1)} = \mathbf{p}^{(k)}$, set $\mathbf{p}_{upp} = \mathbf{p}^{(k+1)}$ and stop. Otherwise, set $k = k+1$ and go to step 2.

A numerical experiment of Algorithm 1 is illustrated in Fig. 2 for the case of $N = 20$, $c = \gamma = 5$ and $\Lambda = 50$. We set $p_0^{(0)} = \gamma/N$ and $p_n^{(0)} = 0$ for $1 \le n \le c-1$. It can be seen that Algorithm 1 iterates only 6 times to find

$$\mathbf{p}_{upp} = [0.08237, 0.06124, 0.04086, 0.02220, 0.00704],$$
$$R_{upp} = 4.1545.$$

VI. A Heuristic Design

As mentioned at the end of Section IV, it is in general infeasible to find exact $\mathbf{p}_{opt}$. To overcome this difficulty, this section proposes a heuristic approach for approximating $\mathbf{p}_{opt}$. Moreover, a method to reduce computation overhead is presented. It will be shown via numerical results that our design enjoys near-optimal performance.

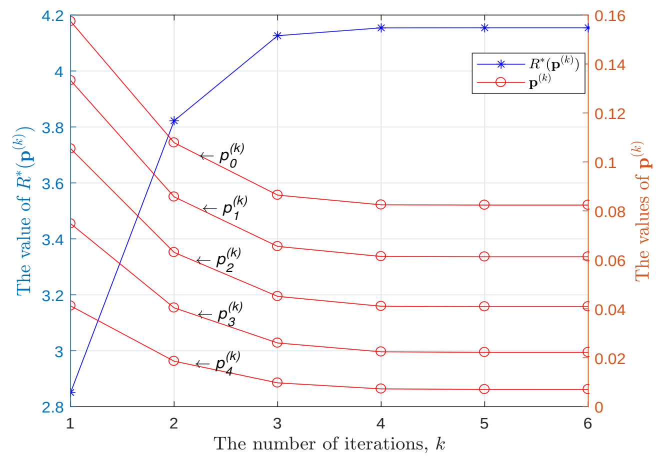


Fig. 2. The iteration procedure of the parameters $\mathbf{p}$ in Algorithm 1 and the corresponding $R^*(\mathbf{p})$ for the case of $N = 20$, $c = \gamma = 5$ and $\Lambda = 50$.

		$N = 10$	$N = 20$	$N = 40$
$c = 2$	$\Lambda = 10$	0.0007440	0.0007612	0.0007767
	$\Lambda = 100$	0.0005350	0.0005022	0.0004913
$c = 3$	$\Lambda = 10$	0.001791	0.001988	0.002074
	$\Lambda = 100$	0.0005975	0.0005659	0.0005.412
$c = 5$	$\Lambda = 10$	0.01945	0.02062	0.02114
	$\Lambda = 100$	0.01113	0.01189	0.01205

TABLE I
The simulative probability that a generic transmission suffers from severe conflict when the system operates with $\mathbf{p}_{upp}$ under $\gamma = 5$.

A. Observation

We first state a simple observation that provides an important clue on our heuristic.

Consider a slot during the lifetime of a given transmission. If there are n ($n < \gamma$) ongoing transmissions in the carrier sensing phase of this slot and more than $\gamma - n$ new transmissions are simultaneously begun at this slot, we say that this given transmission *collides with new transmissions* at this slot. If a given transmission collides with new transmissions at more than one slot, we say this given transmission suffers from *severe conflict.*

In Table I we record the simulative probability that a generic transmission suffers from severe conflict in a variety of cases when the system operates with $\mathbf{p}_{upp}$ under $\gamma = 5$. Each simulation result is obtained from 10 independent simulation runs with 10^7 slots in each run. In all the examined cases, we see that such probability is at most 0.0007767 for $c = 2$, at most 0.002074 for $c = 3$ and at most 0.02114 for $c = 5$. On the other hand, by comparing Eqs. (3), (4), (9), (12) and (13), we know that the scheme with $\mathbf{p}_{upp}$ is in general more aggressive than that with the correct values of $\mathbf{p}_{opt}$. Hence, the following observation holds.

Observation 1. *When the system operates with* $\mathbf{p}$ *whose values are close to* $\mathbf{p}_{opt}$*, there is a small probability that a generic transmission suffers from severe conflict.*

B. Design

Inspired by Observation 1, we modify the parameterized MDP $\mathcal{M}$ to $\mathcal{M}^{**}$ by only redefining the reward at each state. Consider that there are n ongoing transmissions in the carrier sensing phase of a slot and a new transmissions are begun at this slot, i.e., action a is chosen at state n. We determine the reward as follows.

(i) If $n \geq \gamma$, no reward is gained.

(ii) If $n < \gamma$ and $0 \leq a \leq \gamma - n$, the total packet lengths of these a new transmissions are regarded as the positive reward.

(iii) If $n < \gamma$ and $\gamma - n < a \leq N - n$, no positive reward and the total packet lengths of the n ongoing transmissions in the carrier sensing phase are regarded as the negative reward.

Due to the fact that a transmission is completed at the end of a slot with probability $1/\Lambda$, the average packet length of each ongoing transmission in the carrier sensing phase can be calculated as 2Λ and the average packet length of each new transmission can be calculated as Λ. Hence, the reward at state n of $\mathcal{M}^{**}$, denoted by $r_n^{**}(\mathbf{p})$, can be calculated as

$$r_n^{**}(\mathbf{p}) = \Lambda \sum_{a=0}^{\gamma-n} a \cdot \mu_{n,a}(\mathbf{p}) - 2n\Lambda \sum_{a=\gamma-n+1}^{N-n} \mu_{n,a}(\mathbf{p}), \ \forall n \in \mathcal{S}. \tag{19}$$

Then, similar to Eqs. (9) and (13), the long-term average reward of $\mathcal{M}^{**}$, denoted by $R^{**}(\mathbf{p})$, can be calculated as

$$R^{**}(\mathbf{p}) = \sum_{n=0}^{\gamma-1} \pi_n(\mathbf{p}) r_n^{**}(\mathbf{p}). \tag{20}$$

We are interested in $\mathbf{p}_{heu}$ that maximizes $R^{**}(\mathbf{p})$, i.e.,

$$\mathbf{p}_{heu} = \arg\max_{\mathbf{p}\in\mathbb{D}} R^{**}(\mathbf{p}). \tag{21}$$

By comparing Eqs. (12), (13), (19) and (20), we know that the scheme with $\mathbf{p}_{upp}$ is in general more aggressive than that with $\mathbf{p}_{heu}$. Hence, the following observation similar to Observation 1 also holds.

Observation 2. *When the system operates with* $\mathbf{p}$ *whose values are close to* $\mathbf{p}_{heu}$*, there is a small probability that a generic transmission suffers from severe conflict.*

Although the reward at each state calculated in Eq. (19) is different from that calculated in Eq. (4), it is easy to check that $R^{**}(\mathbf{p}) = R(\mathbf{p})$ if we assume that each transmission collides with new transmissions with the same probability and never suffers from severe conflict. So, we propose the following to approximate $\mathbf{p}_{opt}$ based on Observations 1 and 2.

Approximation 1. *When the system operates with* $\mathbf{p}$ *whose values are close to* $\mathbf{p}_{opt}$ *or* $\mathbf{p}_{heu}$*, we have* $R^{**}(\mathbf{p}) \approx R(\mathbf{p})$*. Furthermore, we have* $\mathbf{p}_{heu} \approx \mathbf{p}_{opt}$.

From the description of $\mathcal{M}^{**}$, we see that Algorithm 1 can be applied to $\mathcal{M}^{**}$ to iteratively find $\mathbf{p}_{heu}$. Although the differences are only replacing $r_n^*(\mathbf{p})$ by $r_n^{**}(\mathbf{p})$ and replacing $R^*(\mathbf{p})$ by $R^{**}(\mathbf{p})$ in Eqs. (16)–(18), we summarize the iteration process in Algorithm 2 for the completeness.

Algorithm 2 Policy iteration algorithm to find $\mathbf{p}_{heu}$.

1: **Initialization**. Choose an arbitrary parameter vector $\mathbf{p}^{(0)} \in \mathbb{D}$ as the initial value and set $k = 0$.

2: **Evaluation.** Calculate the relative value $v_n^{**}(\mathbf{p}^{(k)})$ for each $n \in \mathcal{S}$ by the following Bellman equation:

$$\begin{aligned} v_n^{**}(\mathbf{p}^{(k)}) =& r_n^{**}(\mathbf{p}^{(k)}) - R^{**}(\mathbf{p}^{(k)}) \\ &+ \sum_{n'\in\mathcal{S}} \beta_{n,n'}(\mathbf{p}) v_{n'}^{**}(\mathbf{p}^{(k)}), \quad \forall n \in \mathcal{S}. \end{aligned} \tag{22}$$

3: **Improvement.** Update the new parameter as

$$p_n^{(k+1)} = \arg\max_{\widetilde{p_n}\in\mathbb{D}_n} r_n^{**}(\widetilde{p_n}) + \sum_{n'\in\mathcal{S}} \beta_{n,n'}(\widetilde{p_n}) v_{n'}^{**}(\mathbf{p}^{(k)}) \tag{23}$$

for $n = 0, 1, \ldots, c-1$. If there exists $\widetilde{p_n} = p_n^{(k)}$ that attains the maximum in Eq. (23), we set $p_n^{(k+1)} = p_n^{(k)}$.

4: **Stopping Rule.** If $\mathbf{p}^{(k+1)} = \mathbf{p}^{(k)}$, set $\mathbf{p}_{heu} = \mathbf{p}^{(k+1)}$ and stop. Otherwise, set $k = k+1$ and go to step 2.

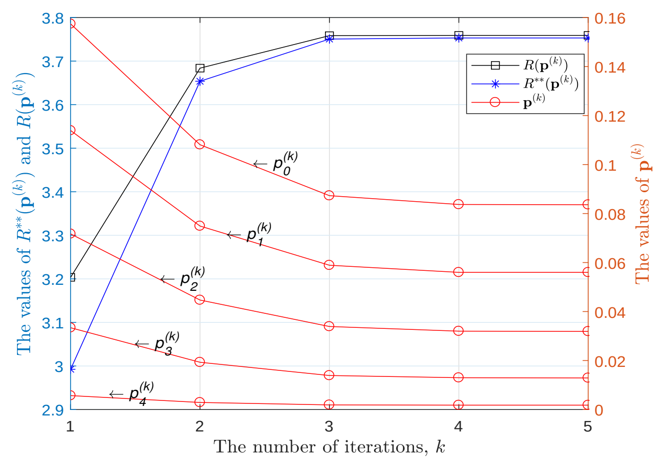


Fig. 3. The iteration procedure of the parameters $\mathbf{p}$ in Algorithm 2 and the corresponding $R^{**}(\mathbf{p})$, $R(\mathbf{p})$ for the case of $N = 20$, $c = \gamma = 5$ and $\Lambda = 50$.

A numerical experiment of applying Algorithm 2 to find $\mathbf{p}_{heu}$ is illustrated in Fig. 3 for $N = 20$, $c = \gamma = 5$ and $\Lambda = 50$. We set $p_0^{(0)} = \gamma/N$ and $p_n^{(0)} = 0$ for $1 \leq n \leq c-1$. It can be seen that only 5 iterations are needed to obtain

$$\begin{aligned} &\mathbf{p}_{heu} = [0.08355, 0.05597, 0.03190, 0.01294, 0.00179], \\ &R^{**}(\mathbf{p}_{heu}) = 3.7531, \qquad R(\mathbf{p}_{heu}) = 3.7590. \end{aligned}$$

On the other hand, we use "GlobalSearch" function included in the MATLAB Global Optimization Toolbox to obtain

$$\begin{aligned} &\mathbf{p}_{opt} \approx [0.08335, 0.05619, 0.03227, 0.01324, 0.00189], \\ &R^{**}(\mathbf{p}_{opt}) \approx 3.7527, \qquad R(\mathbf{p}_{opt}) \approx 3.7594. \end{aligned}$$

These results confirm the accuracy of Approximation 1. More validations will be provided in Section VIII.

Remark 3: When $\gamma = 1$ or $c = 1$, it is easy to see that $r_n(\mathbf{p}) = r_n^*(\mathbf{p}) = r_n^{**}(\mathbf{p})$ for any $n \in \mathcal{S}$. Hence, we have $\mathbf{p}_{opt} = \mathbf{p}_{upp} = \mathbf{p}_{heu}$ for these cases as investigated in [3]–[5], [9], [10], [12].

		$N=10$	$N=20$	$N=40$
$c=2$	$\Lambda=10$	0.0004352	0.0006299	0.0007098
	$\Lambda=100$	0.0004366	0.0005632	0.0006049
$c=3$	$\Lambda=10$	0.0002819	0.0004234	0.0004840
	$\Lambda=100$	0.0001143	0.0001464	0.0001580
$c=5$	$\Lambda=10$	0.0002829	0.0004273	0.0004889
	$\Lambda=100$	0.00005647	0.00007881	0.00008729

TABLE II
The analytical probability that there are more than $\gamma+1$ ongoing transmissions in the carrier sensing phase of a generic slot when the system operates with $\mathbf{p}_{heu}$ under $\gamma=5$.

C. Computation Overhead Reduction

It is well known that policy iteration type algorithms for finding optimal policy of MDP typically converge quickly [24]. However, we see that for each iteration k, Algorithm 2 needs to solve a system of linear equations with $|\mathcal{S}|$ variables to find $\pi_n(\mathbf{p}^{(k)})$ for all $n \in \mathcal{S}$ and to solve another system of linear equations with $|\mathcal{S}|$ variables to find $v_n^{**}(\mathbf{p}^{(k)})$ for all $n \in \mathcal{S}$. Hence, when $|\mathcal{S}| = N+1$ is large, Algorithm 2 involves high computation overhead at each iteration, which is undesirable in practice. To cope with this weakness, we design a simplification method to reduce the state space with almost no performance penalty.

In Table II, we record the analytical probability that there are more than $\gamma+1$ ongoing transmissions in the carrier sensing phase of a generic slot when the system operates with $\mathbf{p}_{heu}$, which can be calculated as $\sum_{n=\gamma+2}^{N} \pi_n(\mathbf{p}_{heu})$ using Eq. (10). Based on these results, we have the following observation.

Observation 3. *When the system operates with $\mathbf{p}$ whose values are close to $\mathbf{p}_{heu}$, there is a small probability that there are more than $\gamma+1$ ongoing transmissions in the carrier sensing phase of a generic slot.*

Inspired by Observation 3, we reduce the state space $\mathcal{S}$ to $\{0, 1, \ldots, \gamma+1\}$ and redefine the state transition probabilities as follows.

$$\beta'_{n,n'}(\mathbf{p}) = \beta_{n,n'}(\mathbf{p}), \qquad \forall n \in \{0,1,\ldots,\gamma+1\}, \; \forall n' \in \{0,1,\ldots,\gamma\},$$

$$\beta'_{n,\gamma+1}(\mathbf{p}) = 1 - \sum_{n'=0}^{\gamma} \beta_{n,n'}(\mathbf{p}), \qquad \forall n \in \{0,1,\ldots,\gamma+1\}.$$

As the MPR capability γ is usually much smaller than the population size N, the computation overhead of Algorithm 2 can be reduced significantly by this simplification.

Based on Observation 3, we propose the following approximation.

Approximation 2. *Let $\widetilde{\mathbf{p}_{heu}}$ denote the solution of Algorithm 2 with the reduced state space. We have $\mathbf{p}_{heu} \approx \widetilde{\mathbf{p}_{heu}}$.*

Still for the case of $N=20$, $c=\gamma=5$ and $\Lambda=50$, we use Algorithm 2 to obtain

$$\widetilde{\mathbf{p}_{heu}} = [0.08402, 0.05619, 0.03198, 0.01296, 0.00179],$$
$$R(\widetilde{\mathbf{p}_{heu}}) = 3.7590.$$

Comparing the above with the results presented at the end of Section VI.B, one sees $\widetilde{\mathbf{p}_{heu}}$ is very close to $\mathbf{p}_{heu}$ and thus there is almost no performance loss. More validations of Approximation 2 will be provided in Section VIII.

Remark 4: Clearly, the heuristic design proposed in this section requires a priori knowledge of the population size N. So, when N is unknown and time-varying, it is essential to enable each user to estimate N at runtime. Many estimation approaches [2]–[4], [10] have been proposed for the traditional p-persistent CSMA under SPR and γ-MPR. Here, we discuss how to apply the estimation approach proposed in [26] to estimate N under the generalized p-persistent CSMA when $c \geq 2$. It was shown in [26] that if $1 \leq a_1 < a_2 \leq N-1-n$,

$$N = \frac{a_2(a_2-a_1)}{a_1 \frac{\chi_{n,a_1}(p_n)\chi_{n,a_2-1}(p_n)}{\chi_{n,a_2}(p_n)\chi_{n,a_1-1}(p_n)} - a_2} + a_2, \tag{24}$$

where $\chi_{n,a}$ denotes the probability that a out of $N-1-n$ silent users actually begin to transmit after carrier sensing n ongoing transmissions. Hence, if the carrier sensing capability $c \geq n + a_2$ for some a_2 and n, $\frac{\chi_{n,a_1}(p_n)\chi_{n,a_2-1}(p_n)}{\chi_{n,a_2}(p_n)\chi_{n,a_1-1}(p_n)}$ is locally measurable by requiring each untransmitting user to perform carrier sensing for the entire slot. This indicates that Eq. (24) provides a linear function for each user to estimate N. We refer our readers to [26] for more details on the estimation algorithm based on Eq. (24). However, a complete study of applying this algorithm to estimate N under the generalized p-persistent CSMA is out of the scope of this paper, and will be an objective of our future work.

VII. Extension to General Models

In our study so far, we have assumed that the reception errors due to wireless fading effect are negligible, and channel coding is not used to protect the packets. These assumptions allow us to provide a nice presentation in Sections IV-VI for approximating $\mathbf{p}_{opt}$. In this section, we relax these assumptions and investigate the throughput performance under more general models.

We consider an "all-or-nothing" MPR model [25], in which the receiver recovers all n signals simultaneously transmitted in a slot with probability ϕ_n and recovers none of them with probability $1-\phi_n$. Here, due to practical constraints, $\phi_n = 0$ if $n > \gamma$. Clearly, the all-or-nothing MPR model contains the γ-MPR model as a special case. In addition, we consider that each user adopts the channel coding with the rate σ.

Let $g_{m,h_1,h_m,u}(\mathbf{p})$ describe the probability of having h_m other ongoing transmissions in the m-th slot of a given transmission and having u unsuccessful slots in the first $m-1$ slots of this transmission, provided that there are h_1 other ongoing transmissions in the first slot of this given transmission. Obviously, $g_{1,h_1,h_1,0}(\mathbf{p}) = 1$. Then, for $m = 2, \ldots, \lambda$, we can recursively obtain

$$g_{m,h_1,h_m,u}(\mathbf{p}) = \sum_{h_{m-1}=0}^{\gamma-1} \phi_{h_{m-1}+1} g_{m-1,h_1,h_{m-1},u}(\mathbf{p}) \xi_{h_{m-1},h_m}(\mathbf{p}) + \sum_{h_{m-1}=0}^{N-1} (1-\phi_{h_{m-1}+1}) g_{m-1,h_1,h_{m-1},u-1}(\mathbf{p}) \xi_{h_{m-1},h_m}(\mathbf{p}). \tag{25}$$

Under the all-or-nothing MPR model with the coding rate σ, we use $q^{\text{AoN}}_{\lambda,h_1}(\mathbf{p})$ to denote the success probability of a λ-slot transmission with h_1 other ongoing transmissions in its first transmission slot. As a λ-slot transmission is successful if and only if there are $\lfloor(1-\sigma)\lambda\rfloor$ or fewer unsuccessful slots in this transmission, we obtain

$$q^{\text{AoN}}_{\lambda,h_1}(\mathbf{p}) = \sum_{h_\lambda=0}^{N-1} \sum_{u=0}^{\lfloor(1-\sigma)\lambda\rfloor-1} g_{\lambda,h_1,h_\lambda,u}(\mathbf{p}) + \sum_{h_\lambda=0}^{\gamma-1} \phi_{h_\lambda+1} g_{\lambda,h_1,h_\lambda,\lfloor(1-\sigma)\lambda\rfloor}(\mathbf{p}). \tag{26}$$

Then, under this scenario, the network throughput, $R^{\text{AoN}}(\mathbf{p})$, can be obtained as the long-term average reward of the parameterized MDP $\mathcal{M}^{\text{AoN}}$, which is modified from $\mathcal{M}$ by only replacing $r_{n,a}(\mathbf{p})$ in (3) with

$$r^{\text{AoN}}_{n,a}(\mathbf{p}) = a\sigma \sum_{\lambda=1}^{\infty} \frac{\lambda}{\Lambda}\left(1-\frac{1}{\Lambda}\right)^{\lambda-1} q^{\text{AoN}}_{\lambda,n+a-1}(\mathbf{p}), \quad \forall n \in \mathcal{S}, \forall a \in \mathcal{A}_n. \tag{27}$$

Although the analysis under the all-or-nothing MPR model with channel coding does not differ from the one under γ-MPR in any fundamental way, we find that it is infeasible to use a similar idea to develop a low-complexity policy for approximating optimal transmission probabilities under the former model. Instead, the robustness of our heuristic design to this model will be investigated in Section VIII.D.

Remark 5: For practical considerations, one would like to analyze the throughput performance under the assumption that the users are heterogeneous in terms of traffic demand, buffer size and carrier sensing capability. It is, in fact, possible to do so by means of a joint state process $(Q^1_t, T^1_t, \ldots, Q^N_t, T^N_t)_{t\in\mathbb{N}}$, where $Q^i_t \in \{0,1,\ldots,B_i\}$ denotes the queue size of user i in the carrier sensing phase of slot t, and $T^i_t \in \{0,1\}$ denotes whether user i is transmitting in the carrier sensing phase of slot t. However, it is extremely difficult or impossible to analyze it because of the curse of dimension. Hence, an appropriate decoupling approximation needs to be further studied, which is out of the scope of this paper.

VIII. Results

This section includes four subsections. To validate the studies in Sections IV-VI, the first subsection compares the analytical and simulative network throughput of our heuristic design with the upper bound, and the second subsection compares our design against the design obtained from "GlobalSearch" included in the MATLAB Global Optimization Toolbox. In the third subsection, the performance improvement of our design over other CSMA-type schemes is shown via simulations. In the fourth subsection, we examine the robustness of our study to the non-geometric distribution of packet lengths, wireless fading effect and employing channel coding, and also examine the accuracy of the analysis in Section VII.

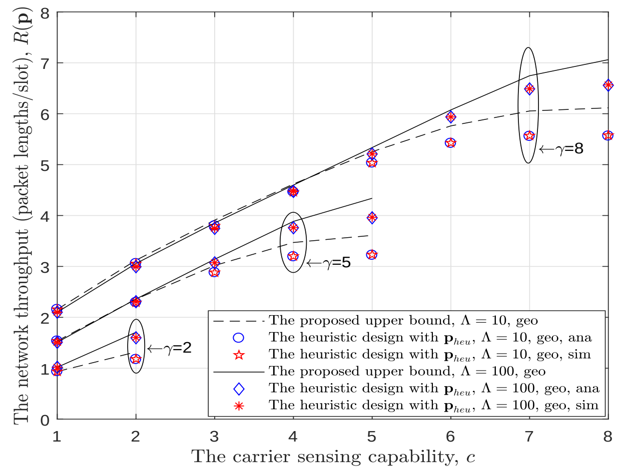


Fig. 4. The network throughput as a function of the carrier sensing capability for different configurations when $N = 20$.

The scenarios considered in the simulations for the first three subsections are in accordance with the descriptions in Sections II, while the scenarios for the fourth subsection will be introduced later. Unless otherwise specified, we set the maximum number of allowed retransmissions to be infinitely large, as it is more interesting to look at the accuracy of our theoretical study in this limiting case. We shall vary the network configuration over a wide range to investigate the impact of system design on the throughput performance. Each simulation result is obtained from 10 independent simulation runs with 10^7 slots in each run.

A. Comparisons with Upper Bounds and Simulation Results

To improve the readability of the figures, the heuristic design with $\widetilde{\mathbf{p}_{heu}}$ is not considered in this subsection, and will be considered in the following subsections.

Fig. 4 shows the analytical and simulative network throughput of our heuristic design with $\mathbf{p}_{heu}$ as a function of the carrier sensing capability c for $\gamma = 2, 5, 8$, $\Lambda = 10, 100$, when N is fixed to 20. The upper bound R_{upp} is also shown for comparison. The curves indicate that our analytical model is very accurate in all the cases. As expected, for each given Λ, both of R_{upp} and $R(\mathbf{p}_{heu})$ become higher as γ or c increases. In particular, for each given pair of γ and Λ, the cases with $c > 1$ significantly outperform the case with $c = 1$, i.e., the optimal traditional p-persistent CSMA considered in [9], [10]. We also observe that for each given pair of γ and Λ, the difference between R_{upp} and $R(\mathbf{p}_{heu})$ is minor when c is small and gradually becomes more noticeable as c increases. This phenomenon is due to the fact that the silent users begin

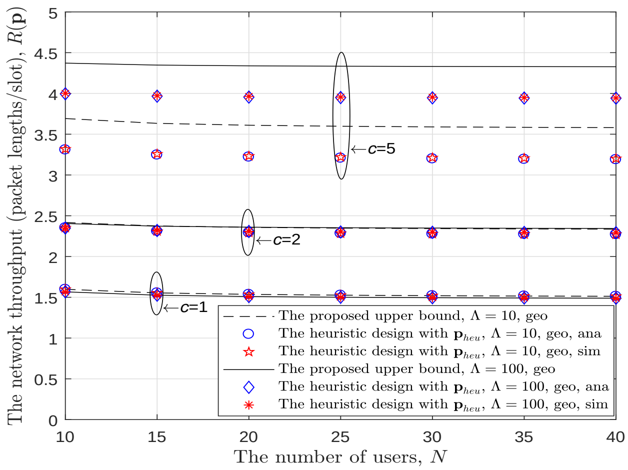


Fig. 5. The network throughput as a function of the number of users for different configurations when $\gamma = 5$.

new transmissions more aggressively as c increases, while R_{upp} is obtained only considering the interference at the first slot of each transmission. In addition, the curves reveal two interesting findings for system design.

(i) An increase of average packet length leads to significant throughput improvement only when the carrier sensing and MPR capabilities are close. The conclusion that an increase of average packet length certainly leads to throughput improvement under SPR [3]–[5] can be seen as a particular case here.

(ii) An increase of MPR capability and a decrease of carrier sensing capability sometimes jointly cause throughput degradation.

The reason of the above is that a decrease of carrier sensing capability results in a negative impact on the utilization of MPR channel, and an increase of average packet length will aggravate this negative impact.

Fig. 5 shows the analytical and simulative network throughput of our heuristic design with $\mathbf{p}_{heu}$ as a function of the number of users N for $c = 1, 2, 5$, $\Lambda = 10, 100$, when γ is fixed to 5. Once again, we see a very good agreement between analytical and simulation results in all the cases. Meanwhile, the observations in Fig. 4 are confirmed again from Fig. 5, especially that an increase of average packet length *cannot* boost the throughput when the carrier sensing capability is relatively weak. The curves further indicate that, for each given pair of Λ and c, the upper bound is almost invariant when N varies. This phenomenon also holds for $R(\mathbf{p}_{heu})$. So, we see that an increase of the number of users incurs almost no performance loss, the same as the corresponding conclusion under SPR [3]–[5].

To show how $R(\mathbf{p}_{heu})$ is close to R_{upp}, Table III records the analytical relative difference $\frac{R_{upp}-R(\mathbf{p}_{heu})}{R_{upp}}$ for $\gamma = 5$, $N = 20$. It can be seen that the relative difference is always zero when $c = 1$, at most 3.389% when $c = 2$, at most 6.491% when $c = 3$, at most 9.274% when $c = 4$, and at most 10.94% when $c = 5$. So, such differences are sufficient to illustrate that our heuristic design has near-optimal performance when c is small, but are insufficient when c is close to γ.

	$\Lambda = 2$	$\Lambda = 5$	$\Lambda = 10$	$\Lambda = 50$	$\Lambda = 100$	$\Lambda = 500$
$c = 1$	0	0	0	0	0	0
$c = 2$	3.389%	3.104%	2.753%	2.304%	2.229%	2.170%
$c = 3$	6.491%	5.775%	4.602%	2.671%	2.248%	1.822%
$c = 4$	8.618%	9.274%	8.034%	4.427%	3.221%	1.495%
$c = 5$	9.097%	10.94%	10.77%	9.520%	8.835%	6.453%

TABLE III
Relative difference $\frac{R_{upp}-R(\mathbf{p}_{heu})}{R_{upp}}$ as a function of the average packet length for different carrier sensing capabilities when $\gamma = 5$, $N = 20$.

B. Comparisons with "GlobalSearch"

To further verify the near-optimal performance of our heuristic design, Table IV presents a comparison among the system that operates with $\mathbf{p}_{heu}$, the system that operates with $\widetilde{\mathbf{p}_{heu}}$, and the system that operates with an approximation of $\mathbf{p}_{opt}$ obtained from "GlobalSearch" for $c = 4$, $\gamma = 5$. The results indicate that these three systems operate with close transmission probabilities, thereby producing almost the same throughput performance. Table V provides the results for $c = \gamma = 5$, which indicate the same observation. We see that the throughput loss is below 0.01% in Table IV and below 0.03% in Table V. As "GlobalSearch" applies to numerically search for global solutions to a problem that contain multiple maxima, these comparisons demonstrate the near-optimal performance of our heuristic design when c is close to γ. Moreover, these comparisons validate the accuracy of Approximations 1 and 2.

It should be noted that applying "GlobalSearch" to $R(\mathbf{p})$ involves quite high computation complexity especially when c or N is large. So, it is prohibitive for a wireless device to use "GlobalSearch" to approximate $\mathbf{p}_{opt}$ in real time.

C. Comparisons with Other CSMA-Type Schemes

To show the performance advantage of our design, we consider the following three asynchronous CSMA-type schemes as benchmarks.

(i) XL-CSMA [12]: each user adopts the transmission probabilities $p_n = \max\left(0, (\gamma^* - n)/(N - n)\right)$ for $n = 0, 1, \ldots, \gamma - 1$, where the tuning parameter γ^* is an integer not larger than γ.

(ii) Threshold-based CSMA/CA [14]: each user maintains a backoff counter with an initialized value from the range $[0, W-1]$, and is required to decrease its backoff counter as soon as the number of sensed ongoing transmissions is below $\max(1, \gamma - 1)$.

(iii) Threshold-based CSMA/CA [15]: each user maintains a backoff counter with an initialized value from the range $[0, W-1]$, and is required to freeze its backoff counter once the number of ongoing transmissions exceeds $\gamma - 1$ and resume decrementing its backoff counter only when the channel becomes idle again.

		p_0	p_1	p_2	p_3	$R(\mathbf{p})$
$N=10$ $\Lambda=10$	"$GlobalSearch$"	0.24711	0.18144	0.11517	0.05300	3.2760
	$\mathbf{p}_{heu}$	0.24744	0.18064	0.11373	0.05156	3.2757
	$\widetilde{\mathbf{p}_{heu}}$	0.24810	0.18099	0.11389	0.05160	3.2757
$N=10$ $\Lambda=100$	"$GlobalSearch$"	0.16468	0.11426	0.06714	0.02778	3.7879
	$\mathbf{p}_{heu}$	0.16611	0.11475	0.06709	0.02757	3.7879
	$\widetilde{\mathbf{p}_{heu}}$	0.16666	0.11503	0.06721	0.02760	3.7879
$N=20$ $\Lambda=10$	"$GlobalSearch$"	0.11219	0.07776	0.04637	0.01995	3.1917
	$\mathbf{p}_{heu}$	0.11221	0.07730	0.04570	0.01935	3.1914
	$\widetilde{\mathbf{p}_{heu}}$	0.11271	0.07753	0.04578	0.01937	3.1914
$N=20$ $\Lambda=100$	"$GlobalSearch$"	0.07236	0.04762	0.02651	0.01033	3.7593
	$\mathbf{p}_{heu}$	0.07270	0.04778	0.02646	0.01024	3.7593
	$\widetilde{\mathbf{p}_{heu}}$	0.07306	0.04795	0.02653	0.01026	3.7593

TABLE IV
A performance comparison among the system operates with $\mathbf{p}_{heu}$, the system operates with $\widetilde{\mathbf{p}_{heu}}$ and the system operates with an approximation of $\mathbf{p}_{opt}$ obtained from "GlobalSearch" for $c=4$, $\gamma=5$.

		p_0	p_1	p_2	p_3	p_4	$R(\mathbf{p})$
$N=10$ $\Lambda=10$	"$GlobalSearch$"	0.24848	0.18278	0.11643	0.05408	0.00862	3.3092
	$\mathbf{p}_{heu}$	0.24832	0.18151	0.11459	0.05236	0.00790	3.3085
	$\widetilde{\mathbf{p}_{heu}}$	0.24899	0.18186	0.11475	0.05240	0.00790	3.3086
$N=10$ $\Lambda=100$	"$GlobalSearch$"	0.16778	0.11659	0.06929	0.02935	0.00447	3.9959
	$\mathbf{p}_{heu}$	0.16761	0.11634	0.06863	0.02876	0.00427	3.9955
	$\widetilde{\mathbf{p}_{heu}}$	0.16819	0.11664	0.06875	0.02879	0.00427	3.9955
$N=20$ $\Lambda=10$	"$GlobalSearch$"	0.11283	0.07834	0.04687	0.02036	0.00304	3.2220
	$\mathbf{p}_{heu}$	0.11260	0.07766	0.04604	0.01965	0.00277	3.2213
	$\widetilde{\mathbf{p}_{heu}}$	0.11311	0.07790	0.04613	0.01967	0.00277	3.2213
$N=20$ $\Lambda=100$	"$GlobalSearch$"	0.07341	0.04862	0.02738	0.01094	0.00156	3.9557
	$\mathbf{p}_{heu}$	0.07339	0.04846	0.02709	0.01071	0.00148	3.9553
	$\widetilde{\mathbf{p}_{heu}}$	0.07377	0.04864	0.02716	0.01072	0.00148	3.9553

TABLE V
A performance comparison among the system operates with $\mathbf{p}_{heu}$, the system operates with $\widetilde{\mathbf{p}_{heu}}$ and the system operates with an approximation of $\mathbf{p}_{opt}$ obtained from "GlobalSearch" for $c=\gamma=5$.

In the comparisons, we use simulations to search for the optimal settings of the above schemes. For practical considerations, the maximum number of allowed retransmissions is set to 4. To improve the readability of the figures, only simulation results are reported in this subsection.

In Fig. 6 (a), we compare our design with optimal XL-CSMA [12] for $N=20$ and $c=\gamma$. Again, we see that $\mathbf{p}_{heu}$ and $\widetilde{\mathbf{p}_{heu}}$ produce almost the same throughput. On the other hand, we observe that our design significantly outperforms optimal XL-CSMA in all the cases especially when Λ is large: 11.4%–40.7% improvement when $\Lambda=10$ and 26.3%–82.2% improvement when $\Lambda=100$. This phenomenon can be attributed to the fact that the setting of transmission probabilities in XL-CSMA is restricted to a special form and does not consider the impact of packet length.

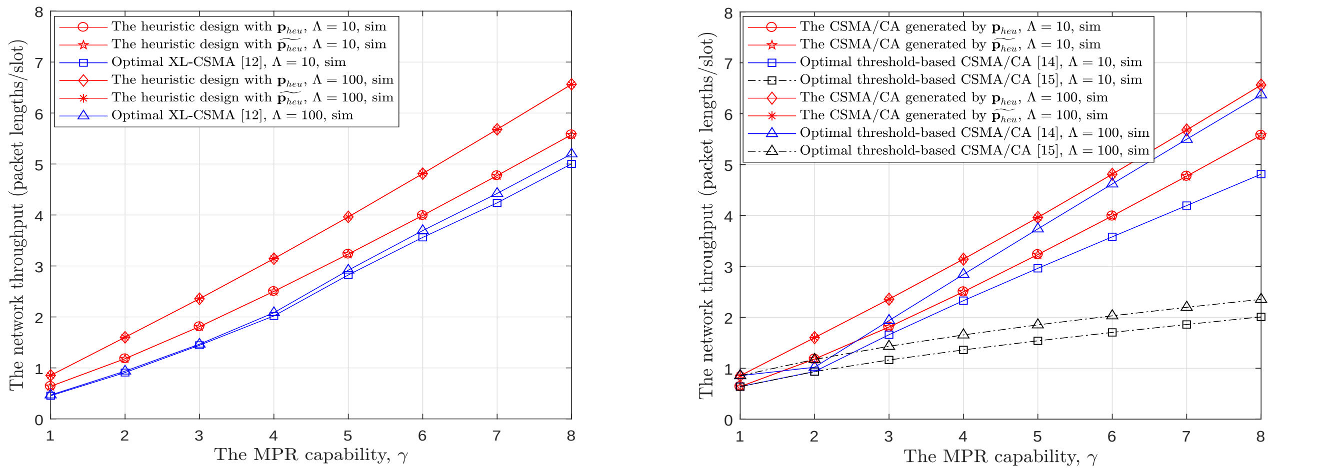
Then, we convert our design to a CSMA/CA scheme with c backoff processes as introduced in Section III, and compare it with the threshold-based CSMA/CA schemes [14], [15] in Fig. 6 (b) for $N=20$ and $c=\gamma$. As expected, when $\gamma=1$, these three schemes have the same performance as they all reduce to the traditional CSMA/CA. When $\gamma>1$, we find that our design outperforms the scheme in [14]: 7.44%–26.3% improvement for $\Lambda=10$ and 3.03%–57.1% improvement for $\Lambda=100$. This happens because the scheme in [14] can be converted to the generalized p-persistent CSMA with $p_0 = p_1 = \cdots = p_{\gamma-2} > 0, p_{\gamma-1}=0$, which has an additional constraint compared with our design. Here, the scheme in [15] obtains the lowest throughput performance.

D. *Robustness of Our design*

1) Robustness to Non-Geometric Distribution of Packet Lengths: In our study so far, we have assumed that the packet lengths follow the geometric distribution, which is reasonable when different types of services are considered. This assumption also allows us to use the Markov systems to model the behavior of generalized p-persistent CSMA. Here, we remove this assumption and examine the robustness of our design to constant-length packets. We still consider the schemes in [12], [14], [15] as benchmarks and use simulations to search for their optimal settings.

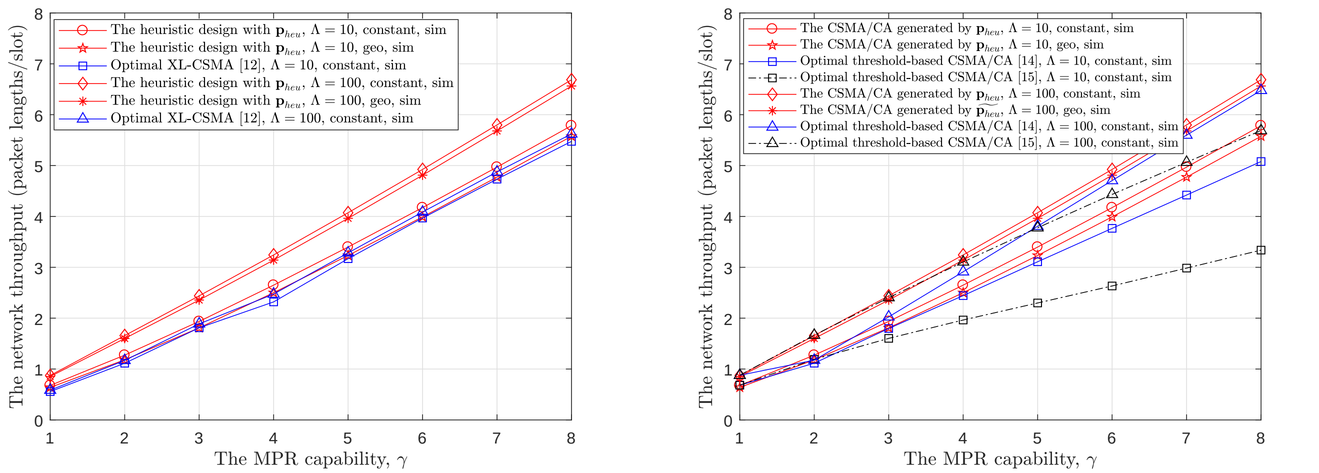
Fig. 7 (a) shows that even under constant-length packets, our design still outperforms the optimal XL-CSMA [12] in all the cases: 5.01%–21.7% improvement when $\Lambda=10$ and 19.0%–49.5% improvement when $\Lambda=100$. However, the performance gap is found smaller than that in Fig. 6 (a). This phenomenon can be attributed to the fact that our design is based on the geometrically distributed packet lengths, and thus may not be near-optimal under constant packet lengths.

Similar results are obtained from Fig. 7 (b), which plots the throughput comparison between our design and threshold-



(a) (b)

Fig. 6. A comparison with XL-CSMA [12] and other asynchronous CSMA/CA [14], [15] for $N = 20$ and $c = \gamma$. The maximum number of allowed retransmissions is set to 4.



(a) (b)

Fig. 7. A comparison with XL-CSMA [12] and other asynchronous CSMA/CA [14], [15] for $N = 20$ and $c = \gamma$. The constant packet lengths and geometrically distributed packet lengths are both considered in the simulation. The maximum number of allowed retransmissions is set to 4.

based CSMA/CA schemes [14], [15] under constant packet lengths. We see that when $\gamma > 1$, our design outperforms the scheme in [14]: 7.75%–14.2% improvement for $\Lambda = 10$ and 3.23%–41.1% improvement for $\Lambda = 100$. Meanwhile, we see that when $\gamma > 1$, our design outperforms the scheme in [15]: 6.97%–73.3% improvement for $\Lambda = 10$ and 1.05%–17.6% improvement for $\Lambda = 100$. The performance gap is also found smaller than that in Fig. 6 (b).

On the other hand, we see from Fig. 7 that the throughput performance of our design under constant packet lengths is slightly better than that under geometrically distributed packet lengths. The improvement is 3.74%–7.84% for $\Lambda = 10$ and 1.87%–3.57% for $\Lambda = 100$. This phenomenon indicates again that our design is effective for constant packet lengths.

2) Robustness to Wireless Fading and Channel Coding: The assumption of negligible wireless fading effect and not using channel coding allows us to provide a nice presentation for approximating $\mathbf{p}_{opt}$. Here, we examine the robustness of our design to the presence of them. To this purpose, we consider successive compute-and-forward (SCF), a signal-processing technique for all-or-nothing MPR, which combines ideas from classical successive interference cancellation and compute-and-forward. We refer our readers to [25] for more details on SCF. With a particular focus on the symmetric rates and complex-valued channel models, some values of ϕ_n under independent Rayleigh-fading environment are listed in Table VI.

Fig. 8 shows the analytical and simulative network throughput of our heuristic design with $\widetilde{\mathbf{p}_{heu}}$ as a function of SNR for the coding rate $\sigma = 4/5, 16/17, 1$, when N is fixed to 20. The results for the designs obtained from "GlobalSearch" are also shown for comparison. We see a discrepancy between analytical and simulative results for the low SNR values and high coding rates, where the analysis is more optimistic. This

SNR (dB)	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
5	0.9839	0.9663	0.9460	0.9176	0.8757
6	0.9926	0.9840	0.9756	0.9644	0.9493
7	0.9966	0.9928	0.9890	0.9846	0.9797
8	0.9984	0.9971	0.9953	0.9936	0.9914
9	0.9993	0.9986	0.9982	0.9973	0.9964
10	0.9997	0.9994	0.9993	0.9988	0.9985

TABLE VI
The values of ϕ_n for SCF-based all-or-nothing MPR under independent Rayleigh-fading environment when the receiver is equipped with 4 antennas, the symmetric message rate is equal to 2, and $\gamma = 5$.

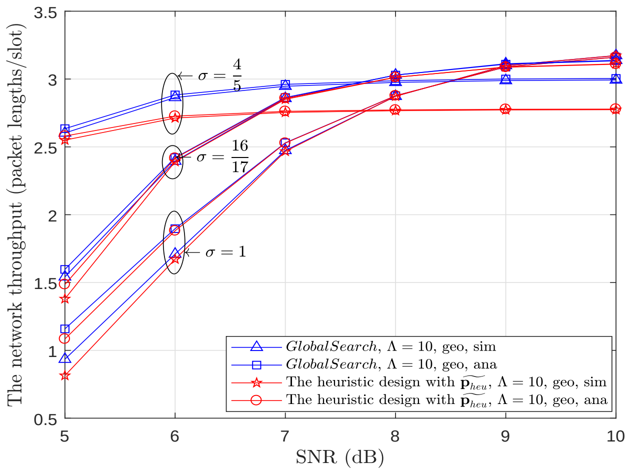


Fig. 8. The network throughput as a function of SNR for different coding rates, when N is fixed to 20. The maximum number of allowed retransmissions is set to 4.

discrepancy is due to the fact that the packets need more retransmissions in these cases especially when the packets have longer lengths, and hence the packet length distribution *cannot* be considered geometric. We also note that the throughput performance under $\widetilde{\mathbf{p}_{heu}}$ is still close to that under "GlobalSearch", but the gap is noticeable when a low coding rate is used. This is because that a lower coding rate allows more unsuccessful slots in a successfully received packet, which makes the heuristic of our design more inapplicable. Furthermore, we observe that, as expected, a lower coding rate is helpful to improve the throughput as the SNR decreases.

In sum, although our design becomes slightly inapplicable in the presence of wireless fading and channel coding, it is still able to offer acceptable throughput performance.

IX. Conclusions

To efficiently utilize the γ-MPR channel, a generalization of traditional p-persistent CSMA and its conversion to CSMA/CA have been proposed. In such CSMA, each user is allowed to adopt different transmission probabilities according to different carrier sensing outcomes. Under the assumption that the packet lengths follow the geometric distribution, a parameterized MDP has been developed to characterize the behavior of such CSMA, and based on a modified version of this MDP a heuristic design of transmission probabilities with low computation complexity has been proposed. Simulation results have shown that our approach is near-optimal for achieving maximum throughput and is able to provide considerable throughput improvement over other CSMA-type schemes. Simulation results also have shown that our approach is robust to constant packet lengths, wireless fading effect and employing channel coding.

As a by-product of our study, we identify some interesting findings for system design. First, an increase of average packet length would lead to significant throughput improvement only when the carrier sensing and MPR capabilities are close. This helps explain why the traditional p-persistent CSMA (i.e., $c = 1$) is not always superior to slotted-ALOHA under γ-MPR, which was observed in [10]. Second, an increase of carrier sensing capability and a decrease of MPR capability may jointly lead to throughput improvement. This is useful to find a better tradeoff between the throughput performance and hardware cost, which is also our ongoing work.

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